

only at x_{i-1} , x_i and x_{i+1} (this discussion is later correctly continued on pages 181–182).

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1. D. N. DE G. ALLEN & R. V. SOUTHWELL, "Relaxation methods applied to determine the motion, in two dimensions, of a viscous fluid past a fixed cylinder," *Quart J. Mech. Appl. Math.*, v. 8, 1955, pp. 129–145.

2. A. M. IL'IN, "Differencing scheme for a differential equation with a small parameter affecting the highest derivative," *Mat. Zametki*, v. 6, 1969, pp. 237–248 = *Math. Notes*, v. 6, 1969, pp. 596–602.

3. R. B. KELLOGG & A. TSAN, "Analysis of some difference approximations for a singular perturbation problem without turning points," *Math. Comp.*, v. 32, 1978, pp. 1025–1039.

4. J. J. H. MILLER, "Sufficient conditions for the convergence, uniformly in epsilon, of a three point difference scheme for a singular perturbation problem," *Numerical Treatment of Differential Equations in Applications* (R. Ansorge and W. Tornig, Eds.), Lecture Notes in Math., vol. 679, Springer-Verlag, Berlin and New York, 1978, pp. 85–91.

25[5.00, 6.30].—R. GLOWINSKI, J. L. LIONS & R. TREMOIERS, *Numerical Analysis of Variational Inequalities*, North-Holland, Amsterdam, 1981, xxx + 776 pp., 23 cm. Price \$109.75, Dfl. 225.—.

This book is really a compilation of three volumes. Chapters 1–3 and Chapters 4–6 are the respective English translations of volumes I and II of the French edition which appeared in 1976. Following these chapters there are six appendices covering material on variational inequalities developed since the publication of the French edition.

Since a review of the French edition appeared in *Math. Comp.*, v. 32, 1978, pp. 313–314, we give only a brief synopsis of the first six chapters and concentrate on the additional material contained in the appendices.

Chapter 1 deals with the general theory of stationary variational inequalities, Chapter 2 with solving the finite dimensional optimization problems which result from the approximation schemes, and Chapter 3 with the specific model problem of elasto-plastic torsion of a cylindrical bar. The problem of a nondifferentiable cost functional is considered in Chapters 4 and 5, with examples such as the steady flow of a Bingham fluid in a cylindrical duct. Chapter 6 contains a discussion of some general approximation schemes for time dependent variational inequalities.

It is the goal of the appendices to treat what the authors consider to be the most important contributions to the subject since the publication of the original French edition. That substantial progress has been made is evidenced by the fact that the appendices comprise about one third of this book.

For example, one important development has been the estimation of approximation errors in connection with the use of finite element approximation schemes. This material is now heavily represented with results for the obstacle problem in Appendix 1, the elasto-plastic torsion problem in Appendix 2, and the steady flow of a Bingham fluid in Appendix 4.

Besides further discussion of topics presented in the earlier edition such as optimization algorithms, the appendices also contain new applications of the ideas of variational inequalities. These include the solution of nonlinear Dirichlet problems, a brief discussion of quasi-variational inequalities, and the numerical simulation of the transonic potential flow of ideal compressible fluids.

With the additional material now included in the present volume, this book is certainly an essential reference for anyone interested in the numerical solution of problems that can be formulated as variational inequalities.

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26[2.05.3].—HERBERT E. SALZER, NORMAN LEVINE & SAUL SERBEN, *Tables for Lagrangian Interpolation Using Chebyshev Points*, manuscript of 54 pages typewritten text + 267 pages of tables, xeroxed and slightly reduced from computer print-out sheets, deposited in the UMT file.

For n -point Lagrangian interpolation for $f(x)$ given at the Chebyshev points $x_{n,i} = -\cos[(2i - 1)\pi/2n]$, $i = 1(1)n$, there are two tables. The first, which is an auxiliary table of $x_{n,i}$ for every n , and $s_{n,i} = \sin[(2i - 1)\pi/2n]$ for the odd values of n , for $n = 2(1)25(5)50(10)100$, to 25 significant figures, is intended primarily for storage in a computer program for calculating the interpolation coefficients in barycentric form. The second, which is the main table, giving the interpolation coefficients themselves, just for $n = 20$, but for $x = -1(0.001)1$, to 20 significant figures, is convenient also for desk calculation with small computers.

The following topics are included in the introductory text: Relation of tables, use of tables for interpolation and quadrature, possible application to equally spaced arguments, advantages in Chebyshev-point interpolation (minimal remainder term, with convergence and stability of coefficients for increasing n), use of tables for Chebyshev economization as an alternative to the methods of C. Lanczos and C. W. Clenshaw, further development of computational methods using interpolation at Chebyshev nodes (especially in numerical integration), description of computation and checking of the tables, and 44 references.

These are some of the more important points in the text which have not been sufficiently noted or emphasized elsewhere in the literature: For practical applications, the advantage in the much smaller upper bound for the classical remainder term is not nearly so important as the *convergence of the interpolation polynomial* as $n \rightarrow \infty$ for the wide class of continuous functions satisfying the Dini-Lipschitz condition in the real interval $[-1, 1]$ (this includes functions with a bounded first derivative which in turn includes analytic functions) *in conjunction with the much smaller interpolation coefficients* (e.g., for $n = 100$ the largest barely exceeds 1, whereas for equal spacing some coefficients exceed 10^{25} ; furthermore, since the sum of the absolute values of the coefficients $\leq 1 + (2/\pi)\ln n$, the factor for total round-off error is < 4). On the basis of the preceding remarks, instead of the global methods of Lanczos which employ the properties of the Chebyshev polynomials to